SUPPLEMENT TO "A FRAMEWORK FOR GEOECONOMICS"

CHRISTOPHER CLAYTON
School of Management, Yale University

MATTEO MAGGIORI
Graduate School of Business, Stanford University

JESSE SCHREGER
Columbia Business School

B.1. FURTHER DETAILS ON STRATEGIES AND SPE IN REPEATED GAME

In the main text we presented the equilibrium starting from exogenous continuation values of the stage game $\nu_i(\mathcal{B}_i)$. To build an SPE we conjecture and verify a value function $\mathcal{V}_i(\mathcal{B}_i)$ of firm i in the repeated game that is non-decreasing in $\mathcal{B}_i$, that is $\mathcal{V}_i(\mathcal{B}_i) \leq \mathcal{V}_i(\mathcal{B}_i')$ if $\mathcal{B}_i \subset \mathcal{B}_i'$.[1] We first construct the value function in an equilibrium without a hegemon, and then extend the construction to an equilibrium with a hegemon. Finally, Appendix B.1.0.0.3 helps to clarify notationally the difference between individual firms and sectors.

We complete construction of a subgame perfect equilibrium (SPE) for firm i by constructing the associated value function $\mathcal{V}_i(\mathcal{B}_i)$ at each set $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$. This construction follows an iterative process (Abreu et al. (1990)). Note that in this paper we build an SPE but do not focus on sustaining the best possible SPE. In each step, the value function $\mathcal{V}_i(\mathcal{B}_i)$ is given as a fixed point of the equation

$$\mathcal{V}_i(\mathcal{B}_i) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) + \beta\mathcal{V}_i(\mathcal{B}_i)$$

$$s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[\mathcal{V}_i(\mathcal{B}_i) - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)).$$

[1] In the SPE that we construct, suppliers that Distrust individual firm i, i.e. $j \notin \mathcal{B}_i$, Reject any positive order. If hypothetically suppliers in $j \notin \mathcal{B}_i$ Accepted a positive order, firm i would still believe that suppliers in j will reject every future order, given $B_{ij} = 0$. Firm i would then Steal from suppliers in j. Hence, suppliers in j Reject the order. For $\theta_{ij} = 0$ this is an assumption given indifference for the suppliers, and otherwise a strict preference.

In this iterative process, the value function constructed in the SPE with no stealing in steps $n = 0, \ldots, N$ is subsequently used as the off-path continuation values of the SPE at step $N + 1$, until the final step with $\mathcal{B}_i = \mathcal{J}_i$ is reached.

We do not restrict to renegotation-proof equilibria in the sense of (for example) Farrell and Maskin (1989), Bernheim and Ray (1989), and Abreu et al. (1993). As highlighted by Fudenberg and Tirole (1991) and Mailath and Samuelson (2006), the practical relevance of this restriction is not obvious since it involves players coordinating onto a more efficient equilibrium, but many economic problems focus attention on inefficient equilibria. We do not impose such requirements both because the coordination required appears unlikely in practice and because we do not want to embed a notion of efficiency in an equilibrium concept, given our paper is fundamentally about an inefficient world economy.

B.1.0.0.1. *Iterative Construction.* Recall that individual firms are atomistic. Therefore, a single firm deviating and being excluded by (a subset of) future suppliers does not require recomputing equilibrium prices and aggregates at that off-path point. That firm simply produces facing the same prices and aggregates as all other firms, but with only a subset of the available inputs.

Since f_i is increasing, concave, and satisfies Inada conditions, then defining

$$\overline{v}_i \equiv \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i)$$

we have $\overline{v}_i < +\infty$. Thus we must have $\mathcal{V}_i(\mathcal{B}_i) \leq \frac{1}{1-\beta}\overline{v}_i$ for all $\mathcal{B}_i$.

That $\mathcal{V}_i(\emptyset) = 0$ follows trivially from $f_i(0, \ell_i, z) = 0$. Consider first an element $\mathcal{B}_i \in \mathcal{S}_i$, so that the continuation value from the stealing action is zero. To construct an SPE, we define for $u \geq 0$ a function $\mathcal{V}_i(\mathcal{B}_i|u)$ as solving

$$\mathcal{V}_i(\mathcal{B}_i|u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \mathcal{V}_i(\mathcal{B}_i|u) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u. \tag{B.1}$$

Since $\overline{v}_i < +\infty$, we can for any $u \geq 0$ define the unique finite value $v_i(u)$ by

$$v_i(u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u$$

Then, $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta} v_i(u)$ is the unique solution to equation (B.1). Therefore, there is an SPE without stealing with value $\mathcal{V}_i(\mathcal{B}_i) = u$ if $\frac{1}{1-\beta} v_i(u) = u$. Consider the function $\Delta(u) = \frac{1}{1-\beta} v_i(u) - u$. Zeros of this function provide values in SPEs with no stealing. First, $\Delta(0) \geq 0$ (which is thus an SPE if it holds with equality).[2] There is also a positive SPE: from the Inada condition, $\Delta'(0+) = +\infty$, and hence $\Delta(\epsilon) > 0$ for sufficiently small ϵ (note if $\Delta(0) > 0$ this is trivial). Likewise since $v_i(u) \leq \overline{v}_i$, then $\Delta(u) < 0$ for $u > \frac{1}{1-\beta}\overline{v}_i$. Hence by continuity, there is at least one positive SPE $u > 0$. Finally, since f_i is concave and $\sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq u$ describes a convex set, then $v_i(u)$ is increasing and concave in u, and hence $\Delta(u)$ is concave. Therefore, there is exactly one positive value of u.

Next consider the iterative construction. Suppose we have constructed, either as SPEs or with reversion, values for all $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)) \backslash \{\mathcal{B}_i\}$. That is we know the continuation values $\mathcal{V}_i(\mathcal{B}_i \backslash S)$ $\forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ from previously constructed SPEs. We can then use these values to construct $\mathcal{V}_i(\mathcal{B}_i)$ as follows. First, for $u \geq 0$ we define

$$\mathcal{V}_i(\mathcal{B}_i|u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \mathcal{V}_i(\mathcal{B}_i|u)$$
$$s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \Big[u - \mathcal{V}_i(\mathcal{B}_i \backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

Thus defining stage game payoff as

$$v_i(u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) \quad s.t. \quad \sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta \Big[u - \mathcal{V}_i(\mathcal{B}_i \backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

then we have $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta} v_i(u)$. We construct the fixed points, if any, of $\frac{1}{1-\beta} v_i(u) = u$. As before, $v_i(u)$ is increasing and concave, and therefore there are at most two positive fixed points (for given continuation values).

If an element $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$ has no SPE associated with no stealing, then we assume that at the beginning of a period in which firm i faces $\mathcal{B}_i$, the suppliers that Trust firm i automatically update to an element $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ such that $\hat{\mathcal{B}}_i$ results in an SPE with no stealing. As a result, $\mathcal{V}_i(\mathcal{B}_i) = \mathcal{V}_i(\hat{\mathcal{B}}_i)$. That is to say, suppliers understand that if the suppliers that

[2] For example, it holds with equality if $\theta_{ij} > 0$ for all j.

Trust firm i were $\mathcal{B}_i$, the firm would in fact Steal from a subset with probability 1, and therefore suppliers update accordingly. We assume throughout the paper that $\mathcal{B}_i = \mathcal{J}_i$ has an SPE with no stealing.

B.1.0.0.2. *Continuation Value Functions in Hegemon Problem.* The hegemon's optimal contract was also characterized for a given set of continuation value functions ν_i. We now provide the equilibrium consistency conditions for a Markov equilibrium. Consider a set of continuation value functions $\nu = \{\nu_i\}$ for firms. Given these continuation value functions, let (Γ, P, z) be the hegemon's optimal contract, prices, and aggregates when the continuation value functions are ν. Then, (Γ, P, z, ν) is an equilibrium if: (i) $\nu_i(\mathcal{B}_i) = \mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)\backslash\{\mathcal{J}_i\}$; and, (ii) $\nu_i(\mathcal{J}_i) = V_i(\Gamma_i)$. To clarify, ν_i are exogenous continuation value functions, $\mathcal{V}_i$ is the off-path value function in the construction of the SPE, and V_i is the equilibrium on-path value function.

B.1.0.0.3. *Individual Firms versus Sectors.* Our paper somewhat abuses notation by identifying i as both a sector of suppliers and an individual firm within that sector. More completely, we could write that sector i has a unit continuum of firms, indexed by $h \in [0, 1]$, and denote $\mathcal{B}_{ih}$ the set of sectors that "Trust" firm ih (i.e., firm h in sector i). The equilibrium would then be determined by the full collection $\mathcal{B} = \{\mathcal{B}_{ih}\}_{i,h}$ and so we could write value functions accordingly. We abused notation as follows. First, we are studying a Markov equilibrium in which all firms will be Trusted on the equilibrium path. In our Nash structure, firm ih chooses its behavior taking as given that all other firms will remain Trusted. If it goes off the equilibrium path by Stealing, it becomes Distrusted by certain sector(s). Since firm ih is infinitesimal, the equilibrium prices P and aggregates z^* do not change once it goes off the equilibrium path (since every other firm, including others within its sector, have chosen not to Steal). Since (P, z^*) are not affected by firm ih's choice, we can write firm ih's value function as $\mathcal{V}_{ih}(\mathcal{B}_{ih})$, leaving implicit the dependence on (the path of) (P, z^*). And since all firms in sector i are identical and we are studying a symmetric equilibrium, we further abuse notation by dropping the h subscript.

B.2. EXTENDING THE FRAMEWORK: HEGEMONIC COMPETITION FOR DOMINANCE

We now consider the possibility that multiple countries are hegemons. For simplicity, we focus on the case in which two countries, m_1 and m_2, are hegemons. To streamline analysis, we focus on competition over transfers, and assume constant prices and no z-externalities (Definitions 1 and 2).

Each hegemon offers a contract as described in Section 3, taking as given the contract offered by the other hegemon. As usual, we begin by taking as given continuation value functions ν_i of firms.

Competition Setup. Let $\mathcal{C} = \mathcal{C}_{m_1} \cup \mathcal{C}_{m_2}$ be the set of firms that contract with at least one hegemon. Hegemon $m \in \{m_1, m_2\}$ offers a contract $\{\Gamma_i^m\}_{i \in \mathcal{C}_m}$, where $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, \tau_i^m\}_{i \in \mathcal{C}_m}$ denotes the contract offered to firm $i \in \mathcal{C}_m$. As in Section 3, the joint threat $\mathcal{S}_i'$ must be feasible. In the analysis that follows, it will be notationally convenient to designate a hypothetical trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ offered by hegemon m to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$. This reduces cumbersome notation of tracking which firms are offered one or two contracts by ensuring all firms in $\mathcal{C}$ are offered two contracts (one of which may be trivial, purely hypothetical, and equivalent to their outside option). We let $\Gamma^m = \{\mathcal{S}_i'^m, \mathcal{T}_i, \tau_i^m\}_{i \in \mathcal{C}}$ be hegemon m's contract, including trivial contracts offered to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$.

Firm i faces revenue-neutral wedges and transfers from both hegemons that are added together when both contracts are accepted.[3] Anticipating that a best response to hegemon $-m$ setting $\tau_i^{-m} = 0$ is for hegemon m to set $\tau_i^m = 0$, we will solve the model assuming all wedges to be zero, and then verify that neither hegemon has an incentive to deviate to nonzero wedges. Therefore, we write the contract $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^{m_1} + \mathcal{T}_i^{m_2}, 0\}$ as the combined contract when firms accept both contracts.

The joint threat $\mathcal{S}_i'$ arising when firm i accepts both contracts is constructed by taking the union of joint trigger sets and applying Lemma 1 (see the proof of Lemma 1 for details on triggers). Here we detail the special case where both hegemons offer maximal joint threats, as indeed they will in equilibrium. Recall that $S_i^{Dm} = \bigcup_{S \in \mathcal{S}_i^{Dm}} S$ and $\overline{\mathcal{S}}_i'^m = \{S_i^{Dm}\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, where we define $S_i^{Dm} = \emptyset$ if $i \notin \mathcal{C}_m$. Then, maximal (combined) joint threats,

[3] Each hegemon takes as given the other hegemon's equilibrium rebates when both contracts are accepted. If firm i chooses to only accept one contract, equilibrium rebates by the hegemon whose contract is accepted are those that maintain revenue neutrality under the single contract, while there are no rebates by the hegemon whose contract was rejected. If neither contract is accepted, there are no rebates.

$\overline{S}_i'$, is given by

$$\overline{S}_i' = (\mathcal{S}_i \backslash (\mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2})) \cup \mathcal{X}_i, \quad \mathcal{X}_i = \begin{cases} \{S_i^{Dm_1}, S_i^{Dm_2}\} & S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset \\ \{S_i^{Dm_1} \cup S_i^{Dm_2}\} & \text{otherwise} \end{cases} \tag{B.2}$$

Intuitively, $\overline{\mathcal{S}}_i'$ combines both hegemon's maximal joint threats into a single maximal joint threat if the two have any common threats. If there are no common threats, the two hegemon's maximal joint threats are separate actions within $\overline{\mathcal{S}}_i'$.

Finally, we define the participation constraints of all firms. In particular, hegemon m's contract is accepted by firm i if

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.3}$$

Both contracts are accepted by firm i if

$$V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}. \tag{B.4}$$

Existence of an Equilibrium. We show existence of an equilibrium in which both hegemons offer maximal joint threats, and both hegemons' contracts are accepted. We then discuss how competition shapes the transfers extracted.

The model with two hegemons has to account for the fact that if hegemon m's contract is rejected by firm i, then hegemon m can no longer use firm i in joint threats.[4] This is important because a best response of hegemon m to a contract Γ^{-m} might involve offering a contract to firm i that leads firm i to reject the contract of hegemon $-m$. To make progress, we restrict the form of the network structure as follows. Let $\mathcal{P} = \{i \in \mathcal{C} \mid V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)\}$ denote the set of firms for which the two hegemons can, possibly only jointly, generate a pressure point. Formally, define hegemon pressure points as **isolated** if: $i \in \mathcal{P} \Rightarrow \mathcal{J}_i \cap \mathcal{P} = \emptyset$. This definition implies that if the two hegemons can generate a pressure point on i, then the two hegemons cannot generate a pressure point on any firm $j \in \mathcal{J}_i$ that is immediately upstream from i. It ensures that two firms with pressure points from the set of hegemons

[4] This was not an issue in the model with a single hegemon because that hegemon always ensured its contract satisfied the participation constraint.

they contract with are not directly linked to one another. Using this condition, we prove that an equilibrium exists in which both hegemons offer maximal joint threats with no wedges.

PROPOSITION 5: *Suppose that hegemon pressure points are isolated. An equilibrium of the model with competition exists in which each hegemon m offers a contract featuring maximal joint threats and no wedges, $\Gamma_i^m = \{\overline{\mathcal{S}}_i^{\prime m}, \overline{T}_i^{m*}, 0\}$, to each $i \in \mathcal{C}_m$. Transfers from all firms $i \notin \mathcal{P}$ are zero. Each firm $i \in \mathcal{C}$ accepts the contract(s) it is offered.*

The proof of Proposition 5 proceeds by constructing transfers $\overline{T}_i^{m*}$ such that each contract Γ_i^m is a best response to contract Γ_i^{-m}, and such that both contracts are accepted, that is $V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2}), V_i(\mathcal{S}_i)\}$. The transfers extracted by each hegemon from a foreign firm $i \notin \mathcal{I}_{m_1} \cup \mathcal{I}_{m_2}$ depend on the degree to which they can provide different threats. In the limit where hegemon threats have no overlap, $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2} = \emptyset$, competition is limited because each hegemon offers a different set of threats. By contrast when threats have full overlap, $\mathcal{S}_i^{Dm_1} = \mathcal{S}_i^{Dm_2}$, the two hegemons offer the same set of threats, and so bid each other down to zero transfers, $\overline{T}_i^m = 0$. In this case, firms receive full surplus from the relationships. This result is reminiscent of the Bertrand paradox, in which two firms competing on prices bid each other down to the perfect competition price. This outcome is also efficient ex post, since all joint threats are supplied and no transfers are extracted.

For a firm that is domestic to hegemon m, that is $i \in \mathcal{I}_m$, it remains optimal for hegemon m to demand no transfers, $\overline{T}_i^{m*} = 0$. Hegemon $-m$ then extracts the largest transfer that leaves firm i indifferent between accepting both contracts and accepting only that of hegemon m: $V_i(\overline{S}_i^{\prime}, \overline{T}_i^{-m*}) = V_i(\overline{\mathcal{S}}_i^{\prime m})$. Thus the joint threats that the firm's own hegemon can provide become that firm's outside option, to which that firm is held by the other hegemon.

B.2.0.1. *Proof of Proposition 5*

Given constant prices and no z externalities (Definitions 1 and 2), the objective function of hegemon m is to maximize its country's wealth level, $w_m = \sum_{i \in \mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \sum_j T_{ij}$. We assume that $\tau_i = 0$ for both hegemons, and then verify that neither hegemon has an incentive to deviate.

Given hegemons do not have a pressure point on firm $i \notin \mathcal{P}$, both hegemons must offer a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ to such firms to avoid having their contract rejected. Since all firms $i \notin \mathcal{P}$ therefore trivially accept the contracts they are offered, given isolated pressure points then the decision problem of each hegemon becomes separable across sectors $i \in \mathcal{P}$. This is due not only to separability of the objective function, but also because the joint threat remains feasible even if some firms in $\mathcal{P}$ reject hegemon m's contract, given that every firm $i \in \mathcal{P}$ has $\mathcal{J}_i \cap \mathcal{P} = \emptyset$ (i.e., $S_i^D \subset \mathcal{J}_i \backslash \mathcal{P}$).

We begin by providing the analog of Proposition 1: both hegemons offer contracts featuring maximal joint threats to all firms $i \in \mathcal{P}$.

LEMMA 2: *Fix a contract Γ^{-m} of hegemon $-m$. Then for all $i \in \mathcal{P}$, it is weakly optimal for hegemon m to offer maximal joint threats, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$.*

Proof of Lemma 2. Fix a contract $\Gamma_i^{-m} = \{\mathcal{S}_i'^{-m}, \mathcal{T}_i^{-m}, 0\}$ of hegemon $-m$. The proof strategy is to show that if a contract $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, 0\}$ is accepted by firm i, then the contract $\Gamma_i^{m\prime} = \{\overline{\mathcal{S}}_i'^m, \mathcal{T}_i^m, 0\}$ is also accepted by firm i. Let $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers Γ_i^m, and $\Gamma_i' = \{\mathcal{S}_i'', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers $\Gamma_i^{m\prime}$. Since the contract Γ_i^m is accepted by firm i, then

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

Since $\overline{\mathcal{S}}_i'^m$ is a joint threat of $\mathcal{S}_i'^m$, then $\mathcal{S}_i''$ is a joint threat of $\mathcal{S}_i'$. Therefore, $V_i(\Gamma_i^{m\prime}) \geq V_i(\Gamma_i^m)$ and $V_i(\Gamma_i') \geq V_i(\Gamma_i)$. Therefore,

$$\max\{V_i(\Gamma_i'), V_i(\Gamma_i^{m\prime})\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\},$$

and hence contract $\Gamma_i^{m\prime}$ is also accepted by firm i. Finally, firm i is weakly better off (which is valued by hegemon m if firm i is domestic). Thus, maximal joint threats is a weak best response, concluding the proof of Lemma 2.

From Lemma 2, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$ is a best response to any contract Γ_i^{-m}, and therefore all transfers of m appear under the joint threat. Thus we will focus on the total transfer $\overline{T}_i^m$ for firms $i \in \mathcal{P}$. The optimal contract for firm i is characterized by Propositions 3 and 8 if only one hegemon contracts with i, so assume $i \in \mathcal{C}_{m_1} \cap \mathcal{C}_{m_2}$.[5]

[5]In the language of Online Appendix B.3.9, firm i is a neutral firm.

Let $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^m, 0\}$ be a candidate optimal contract of hegemon m, and let $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^{m_1}, \overline{T}_i^{m_2}, 0\}$ be the joint contract. We split the remainder of the proof between domestic and foreign firms.

Foreign Firms. Let $i \in \mathcal{P} \backslash (\mathcal{I}_{m_1} \cup \mathcal{I}_{m_2})$ be a firm foreign to both hegemons. We begin with the following intermediate result.

LEMMA 3: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm* i *accepts both contracts if and only if one of the following holds:*

1. *Firm* i *is held to its outside option, with*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.5}$$

2. *Firm* i *exceeds its outside option, with*

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i) \tag{B.6}$$

Proof of Lemma 3. Supposing that both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

First, suppose that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy equation B.5. Then, if either hegemon increased its transfer, the firm would reject both contracts and revert to the outside option. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. By way of contradiction, suppose that $V_i(\Gamma_i) > \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Then, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Again by way of contradiction suppose that (without loss) $V_i(\Gamma_i) = V_i(\Gamma_i^m) > V_i(\Gamma_i^{-m})$. Then again, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore,

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i).$$

Finally, supposing equation B.6 holds, then if either hegemon increased its transfer, the firm would reject its contract and accept only that of the other hegemon. A hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore, neither hegemon deviates, and we have an equilibrium, concluding the proof of Lemma 3.

We use Lemma 3 to construct an equilibrium. Since $i \in \mathcal{P}$, $V_i(\overline{\mathcal{S}}_i^{\prime}) > V_i(\mathcal{S}_i)$. Without loss of generality, let $V_i(\overline{\mathcal{S}}_i^{\prime m}) \geq V_i(\overline{\mathcal{S}}_i^{\prime -m})$. We begin by constructing the minimal transfer $t_0^m \geq 0$ such that $V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m) = V_i(\overline{\mathcal{S}}_i^{\prime -m}, 0)$. Since $\overline{S}_i^{\prime}$ is a joint threat of $\overline{\mathcal{S}}_i^{\prime m}$, then $V_i(\overline{\mathcal{S}}_i^{\prime}, t_0^m) \geq V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m)$. If $V_i(\overline{\mathcal{S}}_i^{\prime}, t_0^m) = V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m)$, then $V_i(\Gamma_i) = V_i(\Gamma_i^m) = V_i(\Gamma_i^{-m})$, and hence either equation (B.5) or (B.6) is satisfied and we have an equilibrium.

Suppose instead $V_i(\overline{\mathcal{S}}_i^{\prime}, t_0^m) > V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m)$. Then, we construct a function $t^{-m}(t)$ by

$$V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m + t) = V_i(\overline{\mathcal{S}}_i^{\prime -m}, t^{-m}(t)).$$

We can construct this function from $t = 0$ to $t = \bar{t}$, where $\bar{t}$ solves $V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m + t) = V_i(\mathcal{S}_i)$ (note it is possible for $\bar{t} = 0$).

Suppose first $\exists t^* \in [0, \bar{t}]$ such that

$$V_i(\overline{\mathcal{S}}_i^{\prime}, t_0^m + t^*, t^{-m}(t^*)) = V_i(\overline{\mathcal{S}}_i^{\prime m}, t_0^m + t^*).$$

Then, equation (B.6) is satisfied if $t^* < \bar{t}$, and equation (B.5) is satisfied if $t^* = \bar{t}$. Therefore, by Lemma 3 we have found an equilibrium.

Suppose instead that no such t^* exists, and therefore $V_i(\overline{\mathcal{S}}_i^{\prime}, t_0^m + \bar{t}, t^{-m}(\bar{t})) > V_i(\mathcal{S}_i)$. Then, define $\overline{T}_i^m$ and $\overline{T}_i^{-m}$ such that $\overline{T}_i^m \geq t_0^m + \bar{t}$, $\overline{T}_i^{-m} \geq t^{-m}(\bar{t})$, and $V_i(\overline{\mathcal{S}}_i^{\prime}, \overline{T}_i^m, \overline{T}_i^{-m}) = V_i(\mathcal{S}_i)$. Then, equation (B.5) is satisfied, and hence we have found an equilibrium.

Therefore, an equilibrium exists as described, assuming both hegemons impose zero wedges. Observe that imposing nonzero wedges cannot increase the value of its objective, and leads to its contract being (weakly) rejected. Thus, zero wedges is a best response of each hegemon, concluding this portion of the proof.

Domestic Firms. Let $i \in \mathcal{P} \cap \mathcal{I}_m$ be a domestic firm of hegemon m.

LEMMA 4: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm* $i \in \mathcal{P} \cap \mathcal{I}_m$ *accepts both contracts if and only if one of the following holds:*

1. *Firm* i *is held to its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.7}$$

2. *Firm* i *exceeds its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.8}$$

Proof of Lemma 4. Supposing that both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted, $V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}$. Moreover, if $\overline{T}_i^m > 0$, hegemon m could reduce its transfer while having its contract accepted, strictly improving welfare. Thus, $\overline{T}_i^m = 0$. Finally, suppose that we have two contracts that satisfy equation B.7 and that $\overline{T}_i^m = 0$. If hegemon $-m$ increased its transfer, then its contract would be rejected. Hegemon m has no incentive to increase its transfer, and so we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that $V_i(\Gamma_i) > V_i(\Gamma_i^m)$. Then, hegemon $-m$ could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = V_i(\Gamma_i^m)$, and therefore $V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}$. If this condition holds, and $\overline{T}_i^m > 0$, then hegemon m could decrease its transfer for its domestic firm without its contract being rejected, and so be strictly better off. Therefore, $\overline{T}_i^m = 0$. Finally, suppose this condition holds and $\overline{T}_i^m = 0$. Then, if hegemon $-m$ increased its transfer, its contract would be rejected. Hegemon m cannot further decrease its transfer. Therefore, neither hegemon deviates, and we have an equilibrium. □

Lemma 4 shows that $\overline{T}_i^m = 0$ in any equilibrium, that is a domestic firm is not charged a transfer by its hegemon. Since $\overline{T}_i^m = 0$, then $V_i(\Gamma_i^{-m}) \leq V_i(\Gamma_i)$. We can construct the transfer of hegemon $-m$ as the solution to $V_i(\overline{\mathcal{S}_i}', \overline{T}_i^{-m}) = V_i(\overline{\mathcal{S}}_i'^m)$. If $V_i(\overline{\mathcal{S}}_i'^m) = V_i(\mathcal{S}_i)$, then equation (B.7) is satisfied and we have an equilibrium. If $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$, then equation (B.8) is satisfied and we have an equilibrium. In both cases, zero wedges is part of an optimal policy for both hegemons. Therefore, we have an equilibrium.

This concludes the proof of Proposition 5.

B.3. ADDITIONAL RESULTS AND DERIVATIONS

B.3.1. *Manipulating the Outside Option*

We show how to extend our setup to allow the hegemon to make threats conditional on a firm rejecting the contract. We think of these threats as being a blunt "do what I say or else..." approach to hegemonic power. These threats amount to manipulating the outside option of targeted entities by threatening to cut off access to the inputs controlled by the hegemon if the contract is rejected. We show that in a repeated game these threats are not as powerful as those that instead increase the inside option, such as the joint threats we study in the main text. The intuition is that, for given continuation values, these threats incentivize the target to accept the hegemon's demands by increasing the cost of not doing so. However, in equilibrium, the continuation values are lowered since the same outside option threats will be made in the future. Low continuation values in turn feed back into the current period problem lowering the target incentives to comply with the hegemon's demands. In this sense, such blunt outside option threats are in part self-defeating and threats that increase the inside option are more powerful.

In addition to specifying a joint threat $\mathcal{S}_i'$, transfers $\mathcal{T}_i$, and wedges τ_i, the hegemon can also impose a *punishment* $\mathcal{P}_i$, which is a restriction that the firm permanently loses access to inputs contained in $\mathcal{P}_i \subset \mathcal{S}_i$ if it rejects the hegemon's contract.[6] As with feasibility of joint threats, it is feasible for the hegemon to use $S \in \mathcal{S}_i$ to form a punishment if $\exists j \in S$ such that $j \in \mathcal{C}_m$.[7] Let $\mathcal{B}_i(\mathcal{P}_i) = \mathcal{J}_i \backslash (\bigcup_{S \in \mathcal{P}_i} S)$ be the set of retained inputs given punishment

[6] It is also straight-forward to allow for the punishment to entail only loss of access for the current period.

[7] For brevity we state punishments as losing access to elements of $\mathcal{S}_i$, but it is easy to extend to allow for punishments of losing access to specific inputs (while potentially retaining access to other inputs of $S \in \mathcal{S}_i$).

$\mathcal{P}_i$. The outside option of firm i is therefore

$$V_i^o(\mathcal{P}_i) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i(\mathcal{P}_i)) + \beta\nu_i(B_i(\mathcal{P}_i))$$
$$s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[\nu_i(B_i(\mathcal{P}_i)) - \nu_i(\mathcal{B}_i(\mathcal{P}_i)\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}_i(\mathcal{B}_i(\mathcal{P}_i))).$$

which leaves implicit that $x_{ij}=0$ for $j\notin\mathcal{B}_i(\mathcal{P}_i)$. The participation constraint is therefore

$$V_i(\Gamma_i) \geq V_i^o(\mathcal{P}_i).$$

Similar in spirit to Proposition 1, the optimal punishment is the one that minimizes the outside option, and we denote this minimal value $\underline{V}_i^o$.[8] The optimal contract can be analyzed as in the baseline model but with with $\underline{V}_i^o$ replacing $V_i(\mathcal{S}_i)$ in the participation constraint.

A one-off threatened punishment at date t is (weakly) effective for the hegemon taking continuation values as given. However, in a Markov equilibrium the threat would be made in every period. For exposition, suppose the participation constraint binds. Lowering the future value of retaining access to the hegemon's inputs therefore lowers the continuation value $\nu_i(\mathcal{J}_i) = V_i^o(\mathcal{P}_i)$, which tightens the participation constraint this period. Therefore, threats of punishment for contract rejection are partially self-defeating. In contrast, with joint threats the hegemon first increases the inside option to $V_i(\overline{\mathcal{S}}_i')$, and then uses demands for costly actions to lower it to $V_i(\mathcal{S}_i)$ up to the participation constraint. Moving the inside option up has a positive feedback into the problem since it makes it less attractive in equilibrium not to deal with the hegemon in the future. This is so especially for friendly firms that might be left some surplus in equilibrium. Even for neutral and unfriendly targets, threats that increase the inside option are particularly powerful for the hegemon by providing a positive feedback loop with future continuation values.

[8]That is, $\underline{V}_i^o = \min_{\mathcal{P}_i\subset\mathcal{S}_i|\mathcal{P}_i \text{ is feasible}} V_i^o(\mathcal{P}_i)$. Although the economically intuitive case is the one in which the outside option is minimized with the threat to cut off as many inputs as possible, translating optimality of a maximal utility punishment into cutting off the most varieties requires that V_i^o be a nonincreasing function, which cannot be guaranteed in general due to incentive problems.

B.3.2. *Specializing the Model to Nested CES Production Functions*

Assume there are only two periods and that in the second period there are no incentive problems (i.e. all θ's are set to zero in the second period). Each sector uses a two-tier nested constant elasticity of substitution (CES) production function. Firm i produces using input vector x_i with length $|\mathcal{J}_i|$ and, for simplicity, no local factors. The inputs are partitioned into bundles, where $\tilde{x} \in \tilde{X}_i$ denotes the varieties of inputs used in a given bundle, and $\tilde{X}_i$ is the set of all bundles. We assume each input only enters one bundle. The production function is then given by:

$$f_i(x_i) = \left(\sum_{\tilde{x} \in \tilde{X}_i} \tilde{\alpha}_{i\tilde{x}} \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{\rho_i}{\chi_{i\tilde{x}}}} \right)^{\frac{\xi_i}{\rho_i}} . \tag{B.9}$$

We allow CES parameters $\chi_{i\tilde{x}}$ to vary across bundles. At time zero, the loss in continuation value arising from stealing variety k is given by:

$$\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i \backslash \{k\}) = -\frac{\xi_i}{1-\xi_i} \frac{1-\rho_i}{\rho_i} \log \left[1 - \Omega_{i\tilde{x}_k} \left(1 - \left(1 - \omega_{ik} \right)^{\frac{1-\chi_{i\tilde{x}_k}}{\chi_{i\tilde{x}_k}} \frac{\rho_i}{1-\rho_i}} \right) \right], \tag{B.10}$$

where $\Omega_{i\tilde{x}_k}$ is the expenditure share of firm i on the bundle that contains input k denoted by $\tilde{x}_k$, and ω_{ik} is the expenditure share on input k within that bundle. We provide a step-by-step derivation of this equation below and definitions of the expenditure shares, but we first provide some intuition.

All else equal, losing varieties with bigger expenditure shares leads to a greater loss. Intuitively, losing inputs that are cheap (low p_k) or are technologically a large fraction of production (i.e. high related α's) increases the loss. Losing a variety k is more costly the closer the production function is to constant returns to scale $\xi \uparrow 1$ because a more scalable production suffers more from one of its inputs being constrained at zero.

To understand the role of substitutability within and across buckets, consider the specific bucket that contains variety k. Fix a within-bundle expenditure share ω_{ik}. If that bucket has a parameter $\chi_{i\tilde{x}_k} \leq 0$ (i.e. more complementarity than Cobb-Douglas), then losing variety k amounts to the same as losing the entire bucket. Intuitively, this occurs because the absence of input k makes strictly positive production from that bucket impossible. For parameters

$\chi_{i\tilde{x}_k} > 0$, the loss decreases the more the varieties are substitutable. A similar logic applies across baskets and is governed by the parameter ρ_i.

This example illustrates the role of "alternatives" in diminishing the value of threats to shut off a firm from a particular input. Intuitively, the existence of closely substitutable inputs or the fact that a particular input accounts for a small expenditure share, decreases this input's strategic value in threats.

Derivation of Equation (B.10). Starting from the nested CES production function in equation (B.9), we first solve the expenditure minimization problem associated with bundle $\tilde{x}$, given by

$$\min \sum_{j\in\tilde{x}} p_j x_{ij} \quad s.t. \quad \left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \geq \overline{x}$$

Letting λ denote the Lagrange multiplier on the production constraint, the FOCs are

$$0 = p_j - \lambda\left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}-1} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}-1} \Rightarrow \left(\frac{p_j}{\alpha_{ij}}\frac{\alpha_{ik}}{p_k}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ij} = x_{ik}$$

Substituting into the production constraint yields

$$\overline{x} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \left(\frac{p_k}{\alpha_{ik}}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ik}.$$

Therefore, the expenditure function is $e_i(p,\overline{x}) = P_{i\tilde{x}}\overline{x}$ where the price index of the consumption basket $\tilde{x}$ is $P_{i\tilde{x}} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$.

The optimization problem over bundles, abusing notation by using $\tilde{x}$ as aggregate consumption of bundle $\tilde{x}$, is

$$\max p_i \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}} - \sum_{\tilde{x}\in\tilde{X}_i} P_{i\tilde{x}}\tilde{x}$$

B.16

This yields FOCs

$$p_i\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}\tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}-1}\alpha_{i\tilde{x}}\xi_i\tilde{x}^{\rho_i-1}=P_{i\tilde{x}}\Rightarrow\tilde{x}=\left(\frac{P_{i\tilde{x}_k}}{P_{i\tilde{x}}}\frac{\alpha_{i\tilde{x}}}{\alpha_{i\tilde{x}_k}}\right)^{\frac{1}{1-\rho_i}}\tilde{x}_k$$

$$\tilde{x}_k=\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i-\rho_i}{\rho_i(1-\xi_i)}}\left(\frac{\alpha_{i\tilde{x}_k}}{P_{i\tilde{x}_k}}\right)^{\frac{1}{1-\rho_i}}(p_i\xi_i)^{\frac{1}{1-\xi_i}}.$$

Therefore, expenditures are

$$\sum_{\tilde{x}}P_{i\tilde{x}}\tilde{x}=(p_i\xi_i)^{\frac{1}{1-\xi_i}}\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}$$

while revenues from production are

$$p_i\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}\tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}}=p_i(p_i\xi_i)^{\frac{\xi_i}{1-\xi_i}}\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

If firm i has all inputs left, we therefore have

$$\nu_i(\mathcal{J}_i)=p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}}-(\xi_i)^{\frac{1}{1-\xi_i}}\right]\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

Off-path, the price index is $P_{i\tilde{x}}(\mathcal{B}_i)=\left(\sum_{j\in\tilde{x}\cap\mathcal{B}_i}\alpha_{ij}^{\frac{1}{1-\overline{\chi}_{i\tilde{x}}}}p_j^{-\frac{\overline{\chi}_{i\tilde{x}}}{1-\overline{\chi}_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$ if firm i has inputs $\mathcal{B}_i$ remaining. Therefore,

$$\nu_i(\mathcal{B}_i)=p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}}-(\xi_i)^{\frac{1}{1-\xi_i}}\right]\left(\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}}\right)^{\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}}.$$

Therefore, we have

$$\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\{k\})$$

$$= \frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(\frac{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i\backslash\{k\})^{-\frac{\rho_i}{1-\rho_i}}}\right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(1-\frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}_k}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}\left[1-\left(\frac{P_{i\tilde{x}_k}(\mathcal{J}_i\backslash\{k\})}{P_{i\tilde{x}_k}(\mathcal{J}_i)}\right)^{-\frac{\rho_i}{1-\rho_i}}\right]\right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(1-\Omega_{i\tilde{x}_k}\left[1-\left(1-\omega_{ik}\right)^{\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}\frac{\rho_i}{1-\rho_i}}\right]\right)$$

given the definitions of expenditure shares, $\Omega_{i\tilde{x}_k} = \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}_k}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}$ and $\omega_{ik} = \frac{\alpha_{ik}^{\frac{1}{1-\chi_{i\tilde{x}_k}}}p_k^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}{\sum_{j\in\tilde{x}_k}\alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}_k}}}p_j^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}$.

In the case of Cobb-Douglas ($\rho = 0$),

$$\log\nu_i(\mathcal{B}_i) = \log\left(p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}}-(\xi_i)^{\frac{1}{1-\xi_i}}\right]\right)-\frac{\xi_i}{1-\xi_i}\sum_{\tilde{x}\in\tilde{X}_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\log P_{i\tilde{x}}(\mathcal{B}_i)$$

For the illustrative empirical work below in Appendix B.3.3, we assume that each nested basket has exactly one hegemon input $|\mathcal{I}_m\cap\tilde{x}| = 1$, and then

$$\sum_{k\in\mathcal{I}_m}[\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\{k\})] = \log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\mathcal{I}_m).$$

Therefore, we can recover $\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\mathcal{I}_m)$ by adding up

$$\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\{k\}) = \frac{\xi_i}{1-\xi_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\left[\log P_{i\tilde{x}}(\mathcal{J}_i\backslash\{k\})-\log P_{i\tilde{x}}(\mathcal{J}_i)\right]$$

$$
\begin{aligned}
&= \frac{\xi_i}{1-\xi_i}\Omega_{i\tilde{x}_k}\log\left(1-\omega_{ik}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}} \\
&= -\frac{\xi_i}{1-\xi_i}\frac{1}{\sigma_{i\tilde{x}}-1}\Omega_{i\tilde{x}_k}\log\left(1-\omega_{ik}\right)
\end{aligned}
$$

which uses that $\sigma_{i\tilde{x}} = \frac{1}{1-\chi_{i\tilde{x}}}$.

B.3.3. *Measuring The Loss in Continuation Value*

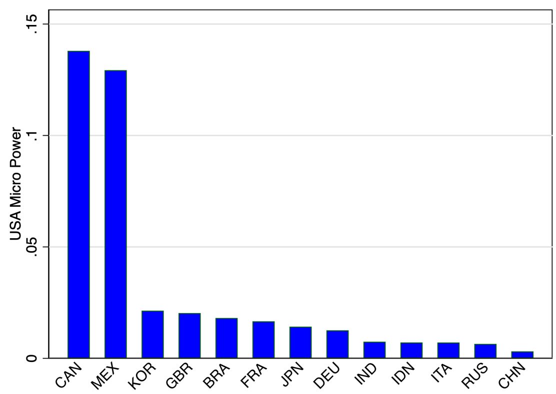
Suppose country n has a representative final goods producer that produces out of domestic and foreign inputs using a nested CES production function as in Appendix B.3.2. From the previous subsection, with a Cobb-Douglas outer nest, we can write the loss to country n's final goods producer from losing access to all the hegemon's goods as

$$
\tilde{\nu}_n \equiv \frac{\xi}{1-\xi}\sum_{k\in\mathcal{I}_m}\left(\log\nu_n(\mathcal{J}_n) - \log\nu_n(\mathcal{J}_n\backslash\{k\})\right) \tag{B.11}
$$

To measure this loss, we start by considering the loss from losing all $k \in \mathcal{I}_m$, that is all firms based in country m, and abstract from threats using firms based outside of m. We set the returns to scale parameter ξ = 0.8, which is within the range of estimates in the literature. We use trade and production data from the OECD Inter-Country Input-Output (ICIO) tables (OECD (2023, http://oe.cd/icio)). We use sectoral elasticities from Fontagné et al. (2022) and restrict our loss calculations to the ICIO goods sectors for which elasticities estimates are available.

Our exercise is in the spirit of Hirschman (1945), evaluating which countries a hegemon has power over based on the nature of the bilateral trade relationship. We focus here on the case where only the hegemon can cut off goods. For every country n, we estimate the loss in continuation value that the United States can cause as in equation B.11. We the estimate losses for the year 2019. While the ICIO data is available until 2020, we use 2019 to avoid effects of Covid on the data. In Figure B.1, we see that the United States has the potential to cause much higher losses to neighbors like Canada and Mexico than to China or Russia. Our measure of the loss of continuation value is related to the Hausmann et al. (2024) estimation of the economic costs that the United States and Europe could impose on Russia via export controls in the Baqaee and Farhi (2022) framework. More generally, our measure parallels the sufficient statistics for welfare gains from international trade in Arkolakis et al. (2012) while focusing on the loss of exports from a single country.

FIGURE B.1.—**USA Ability to Induce Continuation Value Losses in Various Countries, 2019**



Notes: This plots the loss in continuation value calculation following Equation B.11. Trade and production data OECD ICIO tables and trade elasticities from Fontagné et al. (2022).

B.3.4. *Identifying Pressure Points: A Special Case*

In this appendix, we consider an environment in which firms have separable production and provide a necessary and sufficient condition for identifying pressure points. We start by defining the environment:

DEFINITION 3: *The* ***separable production*** *environment assumes that firms that use intermediate inputs have* $f_i(x_i, \ell_i, z) = \sum_{j \in \mathcal{J}_i} f_{ij}(x_{ij}, z)$.

We assume separable production. We write $\Pi_i(x_i, \mathcal{B}_i) = \sum_{j \in \mathcal{B}_i} \pi_{ij}(x_{ij})$, where $\pi_{ij}(x_{ij}) = p_i f_{ij}(x_{ij}, z) - p_j x_{ij}$. Now, suppose that continuation value ν_i is separable across elements of $\mathcal{S}_i(\mathcal{B}_i)$, that is we can write $\nu_i(\mathcal{B}_i) = \sum_{S \in \mathcal{S}_i(\mathcal{B}_i)} v_i(S)$. Then, the incentive constraint associated with $S \in S_i(\mathcal{B}_i)$ is $\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S)$. Therefore, if the incentive constraint holds for $S_1, S_2 \in \mathcal{S}_i(\mathcal{B}_i)$, it also holds for $S_1 \cup S_2$. Thus incentive compatibility with respect to $\mathcal{S}_i(\mathcal{B}_i)$ implies incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Thus the decision problem of firm i becomes separable over elements of the action set $\mathcal{S}_i(\mathcal{B}_i)$, leading to a value function that is separable over elements of the basis, consistent with the assumption.

Now, we move to characterizing pressure points. As a preliminary, the optimization problem of firm i has a corresponding Lagrangian

$$\mathcal{L}(x_i, \lambda | \mathcal{S}_i) \equiv \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{S}_i} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right],$$

where $\lambda_{iS} \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint associated with $S \in \mathcal{S}_i$. We obtain the following result.

PROPOSITION 6: *$S_1, \ldots, S_n \in \mathcal{S}_i$ is a pressure point on firm i if and only if $\lambda_{iS} \neq \lambda_{iS'}$ for some $S, S' \in \{S_1, \ldots, S_n\}$.*

Proposition 6 proves that a necessary and sufficient condition for a pressure point is that the Lagrange multipliers of the existing equilibrium differ among those input relationships that enter the joint threat. To build intuition, return to the example in Figure 1. Consider the equilibrium under individual triggers $\mathcal{S}_i = \{\{j\}, \{k\}\}$, then firms in sector i have a pressure point resulting from the joint threat actions $\{j\}, \{k\}$ if and only if $\lambda_{ij} \neq \lambda_{ik}$. Intuitively, if $\lambda_{ij} > \lambda_{ik}$, then the marginal value of slack in the incentive compatibility constraint for (stealing) good j is higher than for slack in the incentive compatibility constraint for good k. The joint threat creates value by consolidating the two constraints and altering relative production of the two goods, a means of redistributing slack. Heuristically, the joint threat facilitates a *decrease* in production using k in order to create slack that allows for an *increase* in production using j under the joint threat. By contrast if $\lambda_j = \lambda_k$, then slack is equally valuable across goods j and k, even when both multipliers are strictly positive and both constraints bind. In this case, no value is created by forming a joint threat: production under the joint threat is precisely the same as under isolated threats. The proof of Proposition 6 formalizes these intuitions. This result is useful because, in separable production environments, it is possible to identify pressure points based on the existing equilibrium without having to re-compute the firm's optimization problem.

Proof of Proposition 6. We break the proof into the if and only if statements.

If. Suppose that there exist $S', S'' \in \{S_1, \ldots, S_n\}$ such that $\lambda_{iS'} > \lambda_{iS''}$ (without loss of generality). Suppose that we augment the incentive compatibility constraint for S to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S) + \tau_S,$$

where τ_S is a constant (that is set equal to zero in the baseline). Observe that since $S' \cap S'' = \emptyset$, then joint threat constructed from S' and S'' yields the incentive constraint

$$\sum_{j \in S' \cup S''} \theta_{ij} p_j x_{ij} \leq \beta[v_i(S') + v_i(S'')] + \tau_{S'} + \tau_{S'\prime}.$$

Therefore, a weaker expansion of incentive compatible allocations than achieved by a joint threat is to instead increase $\tau_{S'}$ and decrease $\tau_{S''}$ in such a manner that $\tau_{S'} + \tau_{S''} = 0$. If such a perturbation strictly increases value, then creating a joint threat also strictly increases value.

Since $V_i(\mathcal{S}_i, \tau) = \mathcal{L}$, then the welfare effect of a perturbation to τ_S, by Envelope Theorem, is $\frac{\partial V_i}{\partial \tau_S} = \lambda_{iS}$. Therefore, the total profit impact on firm i of the perturbation $d\tau_{S'} = 1$ and $d\tau_{S''} = -1$ is

$$\frac{\partial V_i}{\partial \tau_{S'}} - \frac{\partial V_i}{\partial \tau_{S'\prime}} = \lambda_{iS'} - \lambda_{iS''} > 0.$$

Therefore, there is an $\epsilon > 0$ such that when defining τ by $\tau_{S'} = \epsilon$, $\tau_{S''} = -\epsilon$, and $\tau_S = 0$ otherwise, we have $V_i(\mathcal{S}_i, \tau) > V_i(\mathcal{S}_i, 0)$. But since $V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i, \tau)$, then $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$, and hence $(S_1, \ldots, S_n)$ is a pressure point on i.

Only If. Because the decision problem of firm i is separable across elements of the action set, and because elements $S \notin \{S_1, \ldots, S_n\}$ are unchanged, the same allocations x_{ij}^* for $j \in \bigcup_{S \in \mathcal{S}_i \setminus \{S_1, \ldots, S_n\}} S$ remain optimal. It remains to show that optimal allocations are unchanged for $j \in \bigcup_{S \in \{S_1, \ldots, S_n\}} S$. Suppose first that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} = 0$. Then, x_{ij} is produced at first-best scale, $x_{ij} = x_{ij}^u$. But then since $x_{ij}^* = x_{ij}^u$ is also implementable under joint threats, then the optimal allocation under joint threats is again $x_{ij}^* = x_{ij}^u$, and hence $(S_1, \ldots, S_n)$ is not a pressure point on i.

Suppose next that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} > 0$ and let x_i^* be optimal production under $\mathcal{S}_i$. Because the decision problem of firm i is separable across elements of the action set, let us focus on the subset $\mathcal{X} = \{S_1, \ldots, S_n\}$ of elements in the joint threat. Denoting $\mathcal{L}(x_i, \hat{\lambda}|\mathcal{X})$ the Lagrangian associated with elements $\mathcal{X}$,

$$\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X}) = \sum_{j \in \bigcup_{S \in \mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \hat{\lambda}_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

Recalling that the firm's objective function is concave while each constraint is convex, the Lagrangian has a saddle point at (x_i^*, λ_i).

Next, consider the decision problem of firm i when faced with a joint threat, so that $\mathcal{S}_i'$ has an element $S' = \bigcup_{S \in \mathcal{X}} S$. As again the decision problem of the firm is separable across elements of $\mathcal{S}_i'$, then we can define the Lagrangian of firm i with respect to element S' by

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \mu_{iS'} \left[\beta \sum_{S \in \mathcal{X}} v_i(S) - \sum_{j \in S'} \theta_{ij} p_j x_{ij} \right].$$

Observe that once again, the objective function is concave while the constraint is convex. Since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{X}$, then we can write

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \mu_{iS'} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

Finally, let us define $\mu_{iS'} = \lambda_{iS_1}$. Since $\lambda_{iS_1} = \ldots = \lambda_{i,S_n}$, then we have

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

As a result, we have $\mathcal{L}(x_i, \mu_i|S') = \mathcal{L}(x_i, \lambda_i|\mathcal{X})$ for all x_i. More generally since for any μ_i' there is a corresponding vector $\lambda_{iS}' = \mu_i'$, then since $\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X})$ has a saddle point at (λ_i, x_i^*), then $\mathcal{L}(x_i, \hat{\mu}_i|S')$ has a saddle point at (μ_i, x_i^*). Therefore, x_i^* is also an optimal policy under joint threat $\mathcal{S}_i'$. Therefore, $V_i(\mathcal{S}_i') = V_i(\mathcal{S}_i)$ and hence $(S_1, \ldots, S_n)$ is not a pressure point. This concludes the proof.

B.3.5. *Compliance by Domestic Firms*

Our baseline model assumes that the hegemon contracts with both its own domestic firms and also foreign firms. This means voluntary compliance is also required of domestic firms. Governments typically have much more control over their own firms, through domestic law, than over foreign firms, over which they have no direct legal jurisdiction. Within a domestic economy, power over domestic firms may vary. A hegemon seeking to project geoeconomic power abroad has to find ways to co-opt or coerce its domestic firms into action. There are a number of political economy constraints at home, including legal restrictions, domestic political objectives, interest groups, and other forces that make this domestic power projection less than immediate. This is especially true since private interests of the firms being spurred to action and those of the government might differ. We show how to extend the model so that the hegemon can mandate the choices of its own firms, subject only to a profit positivity constraint.

For the hegemon's domestic firms $i \in \mathcal{I}_m$, we replace the participation constraint (3) with a profit non-negativity constraint, given by

$$V_i(\Gamma_i) \geq 0. \tag{B.12}$$

Relative to the baseline model, there is first a difference in Micro-Power. In particular, the hegemon's Micro-Power over a domestic firm i is now the full value of its operations, $V_i(\Gamma_i)$, rather than just the difference between its inside and outside options, $V_i(\Gamma_i) - V_i(\mathcal{S}_i)$. Intuitively, the hegemon is making the outside option of the firm being shut down completely and having no value. This expands the set of feasible asks of the hegemon of its own firms (all else held equal). Because the hegemon directly values the private profits of its domestic firms, it has no incentive to extract transfers from them, and domestic firms might be left with positive profits (i.e., their non-negativity constraint would not be binding).[9] Second, there is also an effect on the hegemon's optimal asks. In particular, in deriving the

[9] In our repeated game, a domestic firm held to its non-negativity constraint would face added difficulty in sourcing goods because its incentives to deviate would be high in a Markov equilibrium in which it was also held to profit non-negativity in the future. It might therefore be restricted towards purchasing goods that were well enforced by law ($\theta_{ij} = 0$), a calibration likely to apply in particular for many domestic inputs.

optimal contract, equation 6 would be replaced by

$$\varepsilon_{ij}^{z} = \underbrace{\frac{\partial W_m}{\partial w_m}\sum_{k\in\mathcal{I}_m}\frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}}}_{\text{Externalities on Hegemon's Economy}} + \underbrace{\overbrace{\sum_{k\in\mathcal{D}_m}\eta_k\left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]}^{\text{Foreign Firms}} + \overbrace{\sum_{k\in\mathcal{I}_m}\eta_k\frac{\partial \Pi_k}{\partial z_{ij}}}^{\text{Domestic Firms}}}_{\text{Building Power}},$$

and equation 7 (price-based) would be similarly replaced. For any domestic firms held to the profit non-negativity constraint, the hegemon would be incentivized to build power over them by increasing their inside option, $\partial\Pi_k/\partial z_{ij}$, but not by lowering their outside option, which is now fixed at 0.

B.3.6. *Indirect Trade*

In practice, export restrictions have limits as firms find ways around the restriction, a phenomenon referred to as leakage. Restrictions can often be eluded through re-routing trade, for example routing through a third country that simply re-packages the goods. In this appendix, we sketch two ways in which our model can capture or be extended to capture re-routed trade.

B.3.6.0.1. *Re-routed Trade by Introducing Repackaging Sectors.* One way to capture re-routing of trade is through a reinterpretation of some of the productive sectors in our model. To illustrate this possibility, consider the following simple sketch. Consider an individual firm i that can purchase (among others) intermediate goods k and h. Suppose that good h itself is produced out of good k, that is the production technology of sector h is $f_h(x_{hk})$. If sector h is not subject to incentive problems (i.e., $\theta_{hk} = 0$), then optimization yields $p_h f_h'(x_{hk}) = p_k$, and the price of good h is linked to that of k based on the efficiency with which sector h can repackage good k. We think of sector h as being outside the hegemon's control (i.e. cannot be used to make hegemonic threats), while sector k is under the hegemon's control. If goods h and k are substitutable in i's production, then off the equilibrium path when i is cut off by k, firm i is likely to rebalance towards purchasing h, even if h is a costly alternative due to some repackaging and re-routing costs (embedded in h's production function). In this sense, one might expect off-path that indirect trade would be a leading source of rebalancing for a firm that was cut off from direct suppliers of a good.

The above sketch is deliberately simple and illustrative. One could generalize it by having h be a repackager of a set of goods that are similar to k using (for example) a CES technology. One could also assume that firm i uses a CES aggregator to combine good k with the repackaged bundle h.

This form of re-routed trade helps to motivate our assumption on threat feasibility, which restricts joint threats to involve sectors that are at most one step removed from the hegemon. If the hegemon could form threats along longer chains, it could disrupt not only direct but also more indirect trade. We limited the feasibility of threats precise to allow for limited control of economic sectors.

B.3.6.0.2. *Leakage of Threats.* A second possibility is that punishments are imperfectly enforced. In particular, some firms in sector k may continue selling to firm i even when a punishment is supposed to be carried out. We capture this by assuming there is a probability π that firm i continues to be Trusted by suppliers in sectors in S even after it Steals from them and a probability $1-\pi$ that firm i becomes Distrusted. The case $\pi = 0$ is the baseline model. This assumption modifies the incentive constraint (equation 1) of individual firm i to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq (1-\pi)\beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \right].$$

This tightens any given incentive constraint in proportion to the probability the firm is able to evade being cut off.

B.3.7. *Incorporating Military Enforcement*

Political scientists have highlighted different forms of “hard” power (e.g., military threats) and “soft” power (e.g., cultural attraction) (Nye (2004)). The economic threats that we study are certainly softer than military ones and it is interesting to consider how the two types of threats are related. Historically military threats have also been a means of enforcement of private and government relations (see Findlay and O’Rourke (2009) for an historical overview). For example, a hegemon can use a threat of a naval blockade to enforce repayment of sovereign debt or prevent expropriation of a local investment. Both economists (e.g. Bulow and Rogoff (1989)) and political scientists (e.g. Tomz (2012)) have pointed out the limits of military threats in the modern context, but they have surely not disappeared.

We sketch a reduced-form extension of our model to incorporate the military as an enforcement mechanism. We think of the hegemon as a country exogenously endowed with a technology (its military strength) that allows it to serve a role as a global enforcer.[10] In particular, the hegemon is able to directly increase the contract enforceability, that is lower the parameters θ_{ij}, for firms in its economic network. Formally, for each $i \in \mathcal{C}_m$, the hegemon can lower θ_{ij} to $\underline{\theta}_{ij} \leq \theta_{ij}$ for $j \in \mathcal{J}_i$.

The hegemon offers a contract $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i, \tau_i, \theta'_i\}$, and a firm that rejects the contract reverts to the outside option with $(\mathcal{S}_i, \theta_i)$. The value function, parallel to equation (2), is

$$V_i(\Gamma_i) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x^*_{ij}) + T_{ij}] - \sum_{f \in \mathcal{F}_m} \tau^{\ell}_{ij}(\ell_{if} - \ell^*_{if}) + \beta \nu_i(\mathcal{J}_i)$$

$$s.t. \sum_{j \in S} \left[\theta'_{ij} p_j x_{ij} + T_{ij}\right] \leq \beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\right] \quad \forall S \in \Sigma(\mathcal{S}'_i)$$

The participation constraint (equation (3)) is now

$$V_i(\Gamma_i) \geq V_i(\mathcal{S}_i, \theta_i).$$

The hegemon can build Micro-Power through both economic threats and military enforcement. Interestingly, military and economic power can be either complements or substitutes. For example, if military power is able to lower to $\underline{\theta}_{ij} = 0$, then military power is able to obviate the need for economic threats. Similarly, and as highlighted by the BRI example, economic threats can substitute for military enforcement. For example, a joint threat linking a poorly enforced activity (high θ_{ij}) with a well enforced activity (low θ_{ik}) can serve much the same role as directly raising the enforceability of j through a military threat. Finally, it is also clear that these two threats can be complements. Consider a case in which both j and k are similarly poorly enforced, and so a joint threat alone creates

[10]To maintain parallel treatment with economic threats, we think of threats of military enforcement as being cheap in the sense that the hegemon has already paid the sunk cost of having a large military and threats are only carried out off path against specific entities. An important step for future work is to consider the relative costs of the two types of threats, especially incorporating the human and other costs of carrying out military action.

little value. If military enforcement can raise the enforceability of one relationship, say j, then (drawing again on the BRI example) an economic threat that then links the two relationships will help transfer that greater direct enforcement of j to a greater economic enforcement of k through the joint threat.

B.3.8. *Further Characterization of the Equilibrium in the BRI Application*

We provide a solution to the equilibrium and optimal contract in the application to China's Belt and Road Initiative from Section 4.2.[11]

PROPOSITION 7: *The hegemon has a pressure point on entity i if and only if*

$$\theta_{ik} > \frac{\beta}{1-\beta}\frac{1-\xi}{\xi}. \tag{B.13}$$

If the hegemon has a pressure point, then it holds the entity to the participation constraint and sets the transfer to:

$$\overline{T}_i^* = p_i(b^{*\xi} - b^{o\xi}) - R(b^* - b^o),$$

where b^* is the borrowing level under acceptance of the hegemon's contract, and $b^o = \left(\frac{p_i}{R}\frac{\beta}{\theta_{ik}(1-\beta)+\beta}\right)^{\frac{1}{1-\xi}}$ is the borrowing level under a rejection. The borrowing level b^* is the largest value $b^* \leq b^u$ that satisfies

$$p_i b^{*\xi} - (1-\theta_{ik})Rb^* \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - Rb^o\right] + \frac{\beta}{1-\beta}\pi_{ij}^*, \tag{B.14}$$

where $b^u = \left(\frac{p_i\xi}{R}\right)^{\frac{1}{1-\xi}}$ is the optimal unconstrained borrowing level in the stage game and $\pi_{ij}^* = p_i f_{ij}(x_{ij}^*) - p_j x_{ij}^*$ are the equilibrium profits of the manufacturing relationship.

Proposition 7 characterizes a necessary and sufficient condition, the inequality in equation B.13, for the hegemon to have a pressure point. The inequality is intuitive: (i) a pressure point does not exist if discount rates are too low (high β) because, as in a standard Folk

[11] We throughout this application discard the more trivial possible equilibrium in which $b^o = 0$, that is debt is not sustainable under isolated threats because borrowing is not possible in the future.

Theorem argument, enforcement is easy to sustain even on the outside option given high values placed on continuation; (ii) a pressure point does not exist if direct enforcement is too strong (low θ_{ik}) since joint threats have no value if direct enforcement is possible; (iii) if the desired borrowing is too low given the decreasing returns to scale (low ξ).

If there is a pressure point, then Proposition 7 characterizes how powerful this pressure is, that is the transfers extracted, and the borrowing levels on both the inside and outside option. Intuitively, if the manufacturing relationship is very valuable, then under acceptance of the contract the threats are so powerful that the unconstrained level of borrowing b^u can be sustained. Formally, there exists a value $\overline{\pi}_{ij}$ such that $b^* = b^u$ for $\pi_{ij}^* > \overline{\pi}_{ij}$. In this regime, further lowering direct enforcement (increasing θ_{ik}) or increasing discount rates (lowering β), increase the hegemon's power and the equilibrium transfers it extracts. The hegemon's power and transfer also increase in the profitability of the lending relationship (higher p_i or lower R). These comparative statics are intuitive, since both effects lower the outside option level of borrowing b^o but do not affect the inside option borrowing.

If the manufacturing relationship is not sufficiently valuable, $\pi_{ij}^* \leq \overline{\pi}_{ij}$, then b^* is the lower solution to equation B.14 holding with equality. In this regime, the hegemon has a pressure point but the threats are not as powerful, and borrowing occurs at the constrained level (constrained by the incentive compatibility) on both the inside and outside option. In this regime, the comparative statics affect both the inside and outside option level of borrowing, and the net effect on transfers depends on the relative impact on the borrowing.

The above characterization illustrates both the economics of the BRI application and provides a complete solution in terms of the underlying parameters of an application of our general framework. The economics again stresses the role of the hegemon as a global enforcer of economic activity. This role is more powerful the more the activities that could not otherwise be sustained are valuable, the less other means of direct enforcement are present, and the more the hegemon can mix activities with differential enforcement. The conditions that sustain a pressure point and pin down the amount of power are intuitive in terms of the underlying parameters and the economic forces at play.

B.3.8.1. *Proof of Proposition 7*

Denote $\pi_{ik}(b) = p_i b^{\xi} - Rb$. The IC under joint threats is:

$$\theta_{ik} R b + \overline{T}_i \leq \beta \nu_i(\{j,k\}) = \frac{\beta}{1-\beta}\left[\pi_{ik}(b') - \overline{T}'_i + \pi^*_{ij}\right], \tag{B.15}$$

where $'$ variables denote continuation values taken as exogenous in the current period (recall that we are studying a Markov equilibrium, so allocations are identical in all future periods). Since the manufacturing relationship is fully enforced, $\theta_{ij} = 0$, the manufacturing relationship is operated at its first best scale in all periods. The PC in equilibrium is binding by Proposition 8 (see Appendix B.3.9) and given in the current period by:

$$\pi_{ik}(b^*) - \overline{T}^*_i + \pi^*_{ij} + \beta \nu_i(\{j,k\}) = \pi_{ik}(b^o_i) + \pi^*_{ij} + \beta \nu_i(\{j,k\}). \tag{B.16}$$

Simplifying the PC, the current transfer is:

$$\overline{T}^*_i = \pi_{ik}(b^*) - \pi_{ik}(b^o). \tag{B.17}$$

Recall that $\pi_{ik}(b)$, i.e. the firm stage game profit coming from borrowing, is strictly concave and achieves its maximum at

$$b^u = \left(\frac{p_i \xi}{R}\right)^{\frac{1}{1-\xi}},$$

which is the stage game unconstrained optimal level of borrowing. If the entity rejects the hegemon's contract, it operates under isolated threats for the current period, and the contract can be offered again next period. The IC for the current period is:

$$\theta_{ik} R b^o \leq \beta[\nu_i(\{j,k\}) - \frac{1}{1-\beta}\pi^*_{ij}] = \frac{\beta}{1-\beta}\pi_{ik}(b') - \frac{\beta}{1-\beta}\overline{T}'_i. \tag{B.18}$$

Since by Proposition 8 in Online Appendix B.3.9 the participation constraint binds in future periods, we have $\pi_{ik}(b') - \overline{T}'_i = \pi_{ik}(b^{o\prime})$, and therefore the IC constraint after contract rejection simplifies to:

$$\theta_{ik} R b^o \leq \beta \frac{p_i b^{o\prime\xi} - R b^{o\prime}}{1-\beta}. \tag{B.19}$$

B.30

The equilibrium consistency conditions feature $b = b' = b^*$, $b^o = b^{o\prime}$, and $\overline{T}_i = \overline{T}_i' = \overline{T}_i^*$. Substituting in equilibrium consistency and rearranging yields[12]

$$b^o \leq \left(\frac{p_i}{R}\frac{\beta}{\beta + \theta_{ik}(1-\beta)}\right)^{\frac{1}{1-\xi}}$$

An equilibrium therefore features $b^o = b^u$ iff

$$\theta_{ik} \leq \frac{\beta}{1-\beta}\frac{1-\xi}{\xi}.$$

Therefore, there is no pressure point iff the above inequality holds.

Now suppose that we have $\theta_{ik} > \frac{\beta}{1-\beta}\frac{1-\xi}{\xi}$ and there is a pressure point. Under contract rejection, then the binding IC constraint under isolated threats yields as above

$$b^o = \left(\frac{p_i}{R}\frac{\beta}{\theta_{ik}(1-\beta) + \beta}\right)^{\frac{1}{1-\xi}}.$$

Next, under contract acceptance, using the IC and the binding PC we have

$$\theta_{ik} R b^* + \left(p_i(b^{*\xi} - b^{o\xi}) - R(b^* - b^o)\right) \leq \frac{\beta}{1-\beta}\left[p_i b^{o\xi} - R b^o + \pi_{ij}^*\right],$$

which simplifies to:

$$p_i b^{*\xi} - (1-\theta_{ik}) R b^* \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - R b^o\right] + \frac{\beta}{1-\beta}\pi_{ij}^*.$$

The RHS is a constant given the known solution for b^o. Given that $\xi < 1$, the LHS is increasing for $b^* < \left(\frac{\xi p_i}{(1-\theta_{ik})R}\right)^{\frac{1}{1-\xi}}$ and thereafter decreasing. The RHS is maximized at a point above b^u, therefore either the constraint binds or else $b^* = b^u$.[13] Thus we have a

[12] We discard the more trivial possible equilibrium in which $b^o = 0$.

[13] Suppose that in equilibrium we had $b^* > b^u$. Then, $b^* = b^u$ would be implementable for entity i (even if not as part of an equilibrium), a contradiction.

solution at b^u iff

$$p_i b^{u\xi} - (1-\theta_{ik}) R b^u \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - R b^o\right] + \frac{\beta}{1-\beta}\pi^*_{ij},$$

which is a critical threshold on the value of the manufacturing relationship. Therefore, the solution is the largest value $b^* \leq b^u$ that satisfies

$$p_i b^{*\xi} - (1-\theta_{ik}) R b^* \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - R b^o\right] + \frac{\beta}{1-\beta}\pi^*_{ij},$$

completing the proof.

B.3.9. *Classifying Friends and Enemies*

Our framework provides a classification of "friends and enemies" of the hegemon based on externalities. We borrow this terminology from Kleinman et al. (2020) who base it on a country's real income response to a foreign country's increase in productivity. We base it on the externalities that sector has from the hegemon's perspective.[14]

PROPOSITION 8: *If consumer preferences are identical and homothetic (i.e., $U_n(C_n) = U(C_n)$ where U is homothetic), under the hegemon's optimal contract, foreign sector i is:*

1. ***Unfriendly** to the hegemon if $\mathcal{E}_{ij} \leq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j. These sectors are held to their participation constraint and have strictly negative activities $\mathcal{E}_{ij} < 0$ taxed $\tau_{ij} > 0$.*
2. ***Neutral** to the hegemon if $\mathcal{E}_{ij} = 0$ for all $j \in \mathcal{J}_i$. These sectors are held to their participation constraint and are not taxed.*
3. ***Friendly** to the hegemon if $\mathcal{E}_{ij} \geq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j. These sectors have strictly positive activities $\mathcal{E}_{ij} > 0$ subsidized $\tau_{ij} < 0$.*

Proposition 8 delineates three types of relationships: friendly sectors that have only (weakly) positive spillovers from the hegemon's perspective, neutral sectors with no spillovers, and unfriendly sectors with only (weakly) negative spillovers. Sectors can in

[14] The proposition below applies to sectors on which the hegemon has a pressure point.

general have mixed activities, and we leave those sectors unclassified as mixed sectors. A friendly sector i has its strictly positive-externality activities subsidized, while an unfriendly sector has its strictly negative-externality activities taxed. A neutral sector, in contrast, is neither taxed nor subsidized.

The notion of friendship that we develop is both theoretically grounded and relevant for understanding how the hegemon interacts with these sectors in its optimal contract. Friendship can be driven by direct or indirect linkages, and by economic or non-economic motives (the term $\frac{\partial u_m(z)}{\partial z_{ij}}$ in equation (6)). Interestingly, friendship is an important driver of which sectors are held to their participation constraints and achieve no surplus under the optimal contract. Despite the hegemon having all the bargaining power, the hegemon might leave surplus to the foreign sectors (slack participation constraint), but only if it is friendly. Unfriendly and neutral retain no surplus.

These notions can be used to understand the effect of military or political alliances, such as NATO, in the presence of externalities (Olson and Zeckhauser (1966)). We think of these alliances as the hegemon placing positive utility on certain defense sectors of allied countries. In our framework the hegemon would use its global enforcement power to push those allied countries to increase those activities, making them internalize more of the national security externalities, and might also do so at the expense of its own firms' profits or consumers' consumption. Indeed, the hegemon might leave surplus to allied countries' sectors and (optimally) not fully exercise its coercive power on them.

B.3.9.1. *Proof of Proposition 8*

We first show that $\Xi_{mn} = 0$ under identical and homothetic preferences. It suffices to show that excess demand is invariant to wealth transfers between consumers. Excess demand in factor markets does not depend on consumption and so is invariant. Given identical homothetic preferences, we have $C_{ni}(p, w_n) = C_i(p) w_n$ and therefore

$$ED_i = C_i(p) \sum_{n=1}^{N} w_n + \sum_{j \in \mathcal{D}_i} x_{ji} - y_i$$

which is invariant to wealth transfers. Therefore, $\Xi_{mn} = 0$.

Next, we show that the participation constraints of unfriendly and neutral sectors bind. From Proposition 3, the first order condition for transfers yields

$$\overline{\Lambda}_i + \eta_i \geq \frac{\partial W_m}{\partial w_m} > 0$$

and therefore either $\overline{\Lambda}_i > 0$ or $\eta_i > 0$. If $\eta_i > 0$ the proof is completed, so suppose by way of contradiction that $\overline{\Lambda}_i > 0$ but $\eta_i = 0$. Recall that by definition $\overline{\Lambda}_i = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | S_i^D \subset S} \Lambda_{iS}$. Since $\overline{\Lambda}_i > 0$, $\exists S \in \Sigma(\overline{\mathcal{S}}_i')$ such that $S_i^D \subset S$ and $\Lambda_{iS} > 0$. Define $\mathbf{S} = \{S \in \Sigma(\overline{\mathcal{S}}_i') | S_i^D \subset S, \Lambda_{iS} > 0\}$ to be the nonempty set of all such sets. The proof proceeds by considering two mutually exclusive cases.

Case (i). Suppose first that $\forall S \in \mathbf{S}$, $\exists k \in S$ such that $x_{ik}^* > 0$. Let $\mathbf{K} = \{k \in \bigcup_{S \in \mathbf{S}} S \,|\, x_{ik}^* > 0\}$ be the nonempty set of all such k. Returning to the hegemon's Lagrangian in the proof of Proposition 3, consider the perturbation whereby the hegemon increases $\overline{T}_i$ by ϵ arbitrarily small and decreases x_{ik} by $\frac{\epsilon}{p_k \theta_{ik}}$ for all $k \in \mathbf{K}$. Using the derivations of Proposition 3 (i.e., the FOCs for $\overline{T}_i$ and x_{ij} for a foreign firm), the impact of this perturbation on the hegemon's Lagrangian is

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \frac{\partial \mathcal{L}_m}{\partial x_{ik}} = \frac{\partial W_m}{\partial w_m} + \gamma_i - \eta_i - \overline{\Lambda}_i + \Xi_{mn} - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \left[\eta_i \frac{\partial \Pi_i}{\partial x_{ik}} - \overline{\Lambda}_{ik} \theta_{ik} p_k + \mathcal{E}_{ik} \right]$$

where recall that $\overline{\Lambda}_{ij} = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | j \in S} \Lambda_{iS}$. Under the supposition that the participation constraint does not bind ($\eta_i = 0$) and given that $\Xi_{mn} = 0$, we have

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \frac{\partial \mathcal{L}_m}{\partial x_{ik}} = \frac{\partial W_m}{\partial w_m} + \gamma_i - \overline{\Lambda}_i + \sum_{k \in \mathbf{K}} \overline{\Lambda}_{ik} - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \mathcal{E}_{ik}$$

Given the definition of $\mathbf{S}$, we have $\overline{\Lambda}_i = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | S_i^D \subset S} \Lambda_{iS} = \sum_{S \in \mathbf{S}} \Lambda_{iS}$. Since for each $S \in \mathbf{S}$ $\exists k \in \mathbf{K}$ such that $k \in S$, then $\sum_{k \in \mathbf{K}} \overline{\Lambda}_{ik} \geq \sum_{S \in \mathbf{S}} \Lambda_{iS} = \overline{\Lambda}_i > 0$. Therefore,

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \frac{\partial \mathcal{L}_m}{\partial x_{ik}} \geq \frac{\partial W_m}{\partial w_m} + \gamma_i - \sum_{k \in \mathbf{K}} \frac{1}{p_k \theta_{ik}} \mathcal{E}_{ik} \geq \frac{\partial W_m}{\partial w_m} > 0$$

where the second to last inequality follows since firm i is either unfriendly or neutral, $\mathcal{E}_{ik} \leq 0$. But this contradicts that the hegemon's contract was optimal, contradicting the supposition that the participation constraint did not bind in this case.

Case (ii). Suppose instead that $\exists S_0 \in \mathbf{S}$ such that $x^*_{ik} = 0$ for all $k \in S_0$. Since $S_0 \in \mathbf{S}$, then $S_i^D \subset S_0$ and therefore $x^*_{ij} = 0$ for all $j \in S_i^D$. The strategy is to show that the allocation x^*_i is implementable for sector i at its outside option (in which it rejects the contract and faces $\mathcal{S}_i$), without having to make the transfer payment. As such, firm i's value must be higher at the outside option (owing to the Inada condition), contradicting the supposition that the participation constraint did not bind ($\eta_i = 0$).

To show that x^*_i is implementable at the outside option, we must show that

$$\sum_{j \in S} \theta_{ij} p_j x^*_{ij} \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i)$$

Taking $S \in \Sigma(\mathcal{S}_i)$, then since $\mathcal{S}_i^D \subset \mathcal{S}_i$ there exists a $\mathcal{X}_i \subset \mathcal{S}_i^D$ and $\mathcal{Y}_i \subset \mathcal{S}_i \backslash \mathcal{S}_i^D$ such that $S = X_i \cup Y_i$ for $X_i = \bigcup_{Z \in \mathcal{X}_i} Z$ and $Y_i = \bigcup_{Z \in \mathcal{Y}_i} Z$. Since by definition $\overline{\mathcal{S}}_i' \backslash \{S_i^D\} = \mathcal{S}_i \backslash \mathcal{S}_i^D$, then $Y_i \in \Sigma(\overline{\mathcal{S}}_i')$. Because $X_i \subset S_i^D$, $x^*_{ij} = 0$ for $j \in X_i$. Because $Y_i \in \Sigma(\overline{\mathcal{S}}_i')$, because $x^*_{ij} = 0$ for $j \in X_i$, and because x^*_i is IC in the hegemon's problem, we have

$$\sum_{j \in S} \theta_{ij} p_j x^*_{ij} = \sum_{j \in Y_i} \theta_{ij} p_j x^*_{ij} \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash Y_i) \Big] \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash (X_i \cup Y_i)) \Big]$$

where the final inequality follows from monotoncity of ν_i. Therefore, x^*_i is incentive compatible at the outside option, contradicting the supposition that $\eta_i = 0$.

Therefore, the participation constraint binds for unfriendly and neutral sectors ($\eta_i > 0$).

Finally, consider wedges. For an unfriendly or neutral firm, from Proposition 3 we have $\tau^*_{ij} = -\frac{\mathcal{E}_{ij}}{\eta_i}$ and the result follows. For a friendly firm, if the participation constraint binds the result follows analogously. If the participation constraint does not bind, then returning to the argument of the proof of Proposition 3 we have $\tau_{ij} < 0$ for $\mathcal{E}_{ij} > 0$ and small α.

REFERENCES FOR ONLINE APPENDIX

Abreu, Dilip, David Pearce, and Ennio Stacchetti, "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica: Journal of the Econometric Society*, 1990, pp. 1041–1063. []

Abreu, Dilip, David Pearce, and Ennio Stacchetti, "Renegotiation and symmetry in repeated games," *Journal of Economic Theory*, 1993, *60* (2), 217–240. []

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New trade models, same old gains?," *American Economic Review*, 2012, *102* (1), 94–130. []

Baqaee, David Rezza and Emmanuel Farhi, "Networks, Barriers, and Trade," 2022. []

Bernheim, B Douglas and Debraj Ray, "Collective dynamic consistency in repeated games," *Games and economic behavior*, 1989, *1* (4), 295–326. []

Bulow, Jeremy and Kenneth Rogoff, "A constant recontracting model of sovereign debt," *Journal of political Economy*, 1989, *97* (1), 155–178. []

Farrell, Joseph and Eric Maskin, "Renegotiation in repeated games," *Games and economic behavior*, 1989, *1* (4), 327–360. []

Findlay, R. and K.H. O'Rourke, *Power and Plenty: Trade, War, and the World Economy in the Second Millennium* The Princeton Economic History of the Western World, Princeton University Press, 2009. []

Fontagné, Lionel, Houssein Guimbard, and Gianluca Orefice, "Tariff-based product-level trade elasticities," *Journal of International Economics*, 2022, *137*, 103593. []

Fudenberg, D. and J. Tirole, *Game Theory*, MIT Press, 1991. []

Hausmann, Ricardo, Ulrich Schetter, and Muhammed A Yildirim, "On the design of effective sanctions: The case of bans on exports to Russia," *Economic Policy*, 2024, *39* (117), 109–153. []

Hirschman, Albert, *National power and the structure of foreign trade*, Univ of California, 1945. []

Kleinman, Benny, Ernest Liu, and Stephen J Redding, "International friends and enemies," Technical Report, National Bureau of Economic Research 2020. []

Mailath, George and Larry Samuelson, *Repeated games and reputations: long-run relationships*, Oxford university press, 2006. []

Nye, Joseph S, *Soft power: The means to success in world politics*, Public affairs, 2004. []

OECD, "Inter-Country Input-Output tables," 2023, http://oe.cd/icio. []

Olson, Mancur and Richard Zeckhauser, "An economic theory of alliances," *The review of economics and statistics*, 1966, pp. 266–279. []

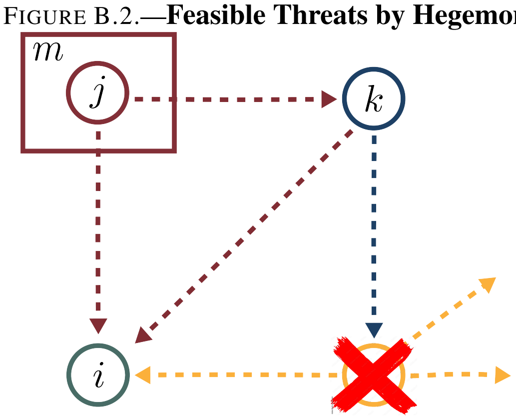
Tomz, M., *Reputation and International Cooperation: Sovereign Debt across Three Centuries* Reputation and International Cooperation: Sovereign Debt Across Three Centuries, Princeton University Press, 2012. []

TABLE B.1

SUMMARY OF NOTATION

Symbol	Meaning	Symbol	Meaning
General Set-up		B_{ij}	Dummy for whether suppliers j Trusts firm i
$\mathcal{I}_n$	Set of sectors in country n. $\mathcal{I}$ set of all sectors	$\mathcal{B}_i$	Set of suppliers that Trust firm i
$\mathcal{F}_n$	Set of factors in country n. $\mathcal{F}$ set of all factors	**Continuation and Value Functions**	
Equilibrium Objects		$\nu_i\left(\mathcal{B}_i\right)$	Exogenous continuation value
p_i	Price of good produced by sector i	$\mathcal{V}_i\left(\mathcal{B}_i\right)$	Eqm value function of firm i in repeated game
p_f^{ℓ}	Price of local factor ℓ	$V_i\left(S_i\right)$	Firm's current value as a function of its action set $\mathcal{S}_i$
p, p^{ℓ}, P	Vector of intermediate goods, factor, all prices	$V_i\left(\Gamma_i\right)$	Value of firm i when accepting contract Γ_i
z	z vector of all externalities z_{ij}	**Hegemon**	
Consumer		D_i	Set of sectors downstream from Sector i
$U_n\left(C_n\right)$	Utility of rep.agent in country n from consumption	$\mathcal{D}_m$	Set of foreign sectors that source at least one input from hegemon's country
$u_n\left(z\right)$	Utility of rep. agent in country n from z	$\mathcal{C}_m$	Set of firms hegemon can contract with
Π_i	Profits of sector i	$\mathcal{J}_{im}$	Set of inputs that sector i sources from sectors in country m
w_n	Income of consumer in country n	T_{ij}	Transfers from i to hegemon in relationship with j. Vector $\mathcal{T}_i$. Sum $\overline{T}_i$
$W_n\left(p, w_n\right)$	Indirect utility function from consumption	τ_{ij}	Revenue-neutral tax i faces on purchases of goods from sector j
Firms		τ_{if}^{ℓ}	Revenue-neutral tax i faces on purchases of factor ℓ
x_{ij}	Intermediate input j used by firm i. Vector x_i	τ_i	Vector of revenue-neutal input and factor taxes faced by i
ℓ_{if}	Local factor $f$used by firm i. Vector ℓ_i	Γ_i	Hegemon's contract $\Gamma_i = \left\{\mathcal{S}_i^{'}, \mathcal{T}_i, \tau_i\right\}$. Vector Γ
y_i	Output of firm i, $y_i = f_i\left(x_i, \ell_i, z\right)$	Ψ^z	Matrix capturing endogenous externality amplification
$\mathcal{J}_i$	Set of suppliers to firm i	$\mathcal{L}^m$	Hegemon's Lagrangian
Stealing		η_i	Lagrange multiplier on the participation constraint of firm i
θ_{ij}	Share of order that can be stolen	Λ_{iS}	Lagrange multiplier on the IC constraint of firm i for action S
a_{ij}	Dummy for whether suppliers j Accepts order of firm i	$\bar{\Lambda}_i$	Sum of multipliers for stealing actions in hegemon's threat
$S_i \subset \mathcal{J}_i$	The subset of sectors from which firm i steals	$\mathcal{E}_{ij}$	Hegemon's perceived externalities from increase in z_{ij}^*
$\mathcal{S}_i$	Action Set: Set of firms' possible stealing decisions	Ξ_{mn}	Hegemon's perceived externalities from a transfer from n to m
$\mathcal{S}_i^{'}$	Joint threat, Coarser partition of $\mathcal{S}_i$	ε_{ij}^{z}	Direct value to hegemon of increasing sector $i's$ use of input j
$\overline{\mathcal{S}}_i'$	Maximal joint threat	ε_{ij}^{zNC}	Indirect value to hegemon of increasing sector $i's$ use of input j
S_i^D	Inputs in hegemon's maximal joint threat	$\varepsilon_j^{P^m}$	Value to hegemon from changes in the price of input j
$P\left(\mathcal{J}_i\right)$	Power set of $\mathcal{J}_i$	Ω_n	Global planner's welfare weight on country n
$\Sigma\left(\mathcal{S}\right)$	Set of all possible unions of elements of $\mathcal{S}$	β	Discount Factor

FIGURE B.2.—**Feasible Threats by Hegemon**



Notes: The figure illustrates the following configuration: sector j is located in the hegemon country and supplies to sector k and i. Sector k supplies to sector i and to another sector (orange and crossed-out), which itself supplies to sector i. The hegemon has a feasible joint threat on sector i via controlling the threats of j and k. The hegemon does NOT have a feasible joint threat on the orange crossed-out sector.